SOLUZIONI

Studio della convergenza

- 01 $\displaystyle\sum_{n=3}^{+\infty} \frac{n^2 - 2n^3 + 2}{(n^2+1)^x}$ $\quad 2x - 3 > 1$, quindi $x > 3/2$
- 02 $\displaystyle\sum_{n=3}^{+\infty} \frac{n^{2-x}}{n^{2x+1}}$ $\quad 3x - 1 > 1$, quindi $x > 2/3$
- 03 $\displaystyle\sum_{n=3}^{+\infty} \frac{n^{2x}}{2^n}$ $\quad \forall x$ reale
- 04 $\displaystyle\sum_{n=3}^{+\infty} \frac{n^{1+x}}{n^{(5x-1)}}$ $\quad 4x - 2 > 1$, quindi $x > 3/4$
- 05 $\displaystyle\sum_{n=3}^{+\infty} \frac{(x-2)^n}{(x+2)^n}$ $\quad |x-2| < |x+2|$, quindi $x > 0$
- 06 $\displaystyle\sum_{n=3}^{+\infty} \frac{(x+2)^n}{(x-3)^{(n-1)}}$ $\quad |x+2| < |x-3|$, quindi $x < 1/2$
- 07 $\displaystyle\sum_{n=3}^{+\infty} \frac{(x-2)^n}{n^2(x+5)^n}$ $\quad |x-2| \leq |x+5|$, quindi $x \geq 3/2$
- 08 $\displaystyle\sum_{n=3}^{+\infty} \frac{n(x+1)^n}{5^{(n-1)}}$ $\quad |x+1| < 5$, quindi $-6 < x < 4$
- 09 $\displaystyle\sum_{n=3}^{+\infty} \frac{(x+1)^n}{n(x-3)^n}$ $\quad -1 \leq \frac{x+1}{x-3} < 1$, quindi $x \leq 1$
- 10 $\displaystyle\sum_{n=3}^{+\infty} \frac{2^{-n}(x+2)^n}{(x-2)^n}$ $\quad |x+2| < 2|x-2|$, quindi $x < 2/3 \ e \ x > 6$
- 11 $\displaystyle\sum_{n=3}^{+\infty} \frac{3^{n+1}(x+2)^n}{5^{(n-2)}(x-2)^n}$ $\quad 3|x+2| < 5|x-2|$, quindi $x < 1/2 \ e \ x > 8$
- 12 $\displaystyle\sum_{n=3}^{+\infty} \frac{(2x+2)^n}{(3x-2)^n}$ $\quad |2x+2| < |3x-2|$, quindi $x < 0 \ e \ x > 4$

- 13 $\sum_{n=3}^{+\infty} \frac{(n^2+n)(x-1)^n}{(n^3)(x+2)^n}$ $\quad -1 \le \frac{x-1}{x+2} < 1$, quindi $x \ge -1/2$
- 14 $\sum_{n=3}^{+\infty} \frac{n^{(2x+5)}}{(n+1)^{(x-2)}}$ $\quad -x-7 > 1$, quindi $x < -8$
- 15 $\sum_{n=3}^{+\infty} \frac{(n+1)^{(2x+4)}}{n^{(7x)}}$ $\quad 5x-4 > 1$, quindi $x > 1$

Calcolo della somma

- 01 $\sum_{n=1}^{\infty} \frac{\pi^n}{n!}$ $\quad S = e^{\pi} - 1$
- 02 $\sum_{n=0}^{\infty} \frac{\pi^{2n}}{n!}$ $\quad S = e^{\pi^2}$
- 03 $\sum_{n=2}^{\infty} \frac{(-1)^n \pi^n}{n!}$ $\quad S = e^{-\pi} - 1 + \pi$
- 04 $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!}$ $\quad S = \cos(\pi) = -1$
- 05 $\sum_{n=0}^{\infty} \frac{(-1)^{(n+1)} \pi^{2n}}{(2n)!}$ $\quad S = -\cos(\pi) = 1$
- 06 $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{(2n+1)}}{(2n+1)!}$ $\quad S = \sin(\pi) = 0$
- 07 $\sum_{n=1}^{\infty} \frac{(-1)^{(n-1)} \pi^{(2n+1)}}{(2n+1)!}$ $\quad S = -(\sin(\pi) - (\pi)) = \pi$ (muggiti di disappunto)
- 08 $\sum_{n=0}^{\infty} \frac{(-1)^{(2n)} \pi^{(2n+1)}}{(2n+1)!}$ $\quad S = \sinh(\pi)$
- 09 $\sum_{n=1}^{\infty} \frac{(-1)^{(2n+1)} \pi^{(2n)}}{(2n)!}$ $\quad S = -(\cosh(\pi) - 1) = 1 - \cosh(\pi)$
- 10 $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ $\quad S = \frac{1}{1-(-1/2)} = \frac{2}{3}$

- 11 $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2^{n+1}}$ $\qquad S = \dfrac{1}{1-(-1/2)} - 1 = -\dfrac{1}{3}$
- 12 $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{2^{n}}$ $\qquad S = \dfrac{1}{1-1/2} - 1 = 1$
- 13 $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n\,2^{n}}$ $\qquad S = -\log(1+\dfrac{1}{2}) = \log(\dfrac{2}{3})$
- 14 $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n}}{(2n+1)\,2^{(2n+1)}}$ $\qquad S = \arctan(\dfrac{1}{2})$
- 15 $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{(n+1)}}{n\,2^{n}}$ $\qquad S = \log(1+\dfrac{1}{2}) - \dfrac{1}{2} = \log(\dfrac{3}{2}) - \dfrac{1}{2}$