

Advanced Mathematical Methods for Engineers -
February 2 2016

1. Determine the general solution of the linear homogeneous system

$$\dot{z} = \mathbb{A}z, \quad \text{where} \quad \mathbb{A} = \begin{bmatrix} 0 & 0 & -2 \\ -1 & 1 & -2 \\ 1 & 0 & 3 \end{bmatrix}.$$

2. Using the separation-of-variable method, determine the solution u of the following problem and discuss its regularity

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 & x \in]0, \pi[, t > 0, \\ u(x, 0) = 3 \sin^2 x & x \in [0, \pi], \\ \frac{\partial u}{\partial t}(x, 0) = 0 & x \in [0, \pi], \\ u(0, t) = u(\pi, t) = 0 & t \geq 0. \end{cases}$$

3. Consider the Cauchy Problem

$$\begin{cases} y' = 1 + \sin^2 y \\ y(x_o) = y_o, \end{cases} \quad (x_o, y_o) \in \mathbb{R}^2.$$

Determine the main properties of its solution and draw a qualitative graph, as (x_o, y_o) ranges in \mathbb{R}^2 .

4. Solve in $\mathcal{D}'(\mathbb{R})$ the equation

$$(x^2 - 9) u' = \delta'(x - 3) + 1.$$