Advanced Mathematical Methods for Engineers - February 3 2015

1. Determine the general solution of the linear homogeneous system

$$\underline{z}' = \mathbb{A}\underline{z}, \qquad \text{where} \qquad \mathbb{A} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}.$$

2. Consider the Cauchy Problem

$$\begin{cases} y' = \dfrac{y^3}{1+x^2} \\ y(x_o) = y_o, \end{cases} \qquad (x_o, y_o) \in \mathbb{R}^2.$$

Determine the main properties of the solution and draw its qualitative graph, as (x_o, y_o) ranges in $\mathbb{R}^2$.

3. Using the method of separation of variables, determine the solution of the following Initial-Boundary Value Problem

$$\begin{cases} u_{tt} - u_{xx} = 0, \qquad x \in [0,1], \quad t \geq 0, \\ u(0,t) = u(1,t) = 0, \qquad t \geq 0, \\ u(x,0) = x^3(1-x)^3, \qquad x \in [0,1], \\ u_t(x,0) = 0, \qquad x \in [0,1]. \end{cases}$$

Discuss the regularity of u.

4. Determine all the solutions $u \in \mathcal{D}'(\mathbb{R})$ of the equation

$$(x^3 - 1)\, u' = \delta'.$$