

# Advanced Mathematical Methods for Engineers - July 7 2016

1. Determine the general solution of the linear homogeneous system

$$\dot{\underline{z}} = \mathbb{A}\underline{z}, \quad \text{where} \quad \mathbb{A} = \begin{bmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ -2 & -1 & 0 \end{bmatrix}.$$

2. Consider the sequence  $f_n : [-10, 10] \rightarrow \mathbb{R}$  defined by

$$f_n(x) = \frac{x}{1 + nx^2}.$$

Compute the limit of the sequence  $\{f_n\}$  in  $C^0([-10, 10])$  endowed with the supremum norm, and in  $L^2(-10, 10)$  endowed with its natural integral norm.

3. Consider the Cauchy Problem

$$\begin{cases} y' = xy e^{-y^2}, \\ y(0) = y_0. \end{cases}$$

Determine the main properties of its general solution and draw a qualitative graph, as  $y_0$  ranges in  $\mathbb{R}$ .

4. Prove that

$$u = e^{-x^2} \left( \text{pv} \frac{1}{x} \right)$$

is a tempered distribution and compute its Fourier transform, justifying all the steps.