Advanced Mathematical Methods for Engineers - July 9 2015

1. Determine the general solution of the linear homogeneous system

$$\underline{z}' = \mathbb{A}\underline{z}, \qquad \text{where} \qquad \mathbb{A} = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{bmatrix}.$$

2. Consider the Cauchy Problem

$$\begin{cases} y' = y \sin y \\ y(x_o) = y_o, \end{cases} \qquad (x_o, y_o) \in \mathbb{R}^2.$$

   Determine the main properties of its solution and draw a qualitative graph, as (x_o, y_o) ranges in $\mathbb{R}^2$.

3. Compute

$$\lim_{n \to +\infty} \left[\frac{2nx^{2n-1}}{x^{4n}+1} + n\cos(nx) \right],$$

   in the sense of $\mathcal{D}'(\mathbb{R})$.

4. Consider the space $H^1(-1,1) = \{f \in L^2(-1,1) \text{ s.t. } f' \in L^2(-1,1)\}$, which is a Hilbert space endowed with the inner product

$$(f,g)_{H^1} = \int_{-1}^{1} f(x)\overline{g(x)}\,dx + \int_{-1}^{1} f'(x)\overline{g'(x)}\,dx.$$

   Let $Z \subset H^1(-1,1)$ be the linear manifold generated by the vectors

$$v_1 = 3, \qquad v_2 = 2x, \qquad v_3 = 4x^2.$$

   In Z build an orthonormal system and approximate $f(x) = e^x$ in Z with the least mean square error.