

Advanced Mathematical Methods for Engineers - September 17 2015

1. Determine the general solution of the linear homogeneous system

$$\underline{z}' = \mathbb{A}\underline{z}, \quad \text{where} \quad \mathbb{A} = \begin{bmatrix} 2 & -13 \\ 2 & 12 \end{bmatrix}.$$

2. Consider the Cauchy Problem

$$\begin{cases} y' = \frac{e^{\frac{1}{y}}}{y-2} & (x_o, y_o) \in \{(x, y) \in \mathbb{R}^2 : y \neq 0, y \neq 2\}. \\ y(x_o) = y_o, \end{cases}$$

Determine the main properties of its solution and draw a qualitative graph, as (x_o, y_o) ranges in $\{(x, y) \in \mathbb{R}^2 : y \neq 0, y \neq 2\}$.

3. Consider

$$u = x - \arctan x.$$

- Prove that $u \in \mathcal{S}'(\mathbb{R})$.
- Compute its Fourier transform \hat{u} .

4. Compute

$$\inf_{a, b \in \mathbb{R}} \int_{-1}^1 |\cos x - (a + bx)|^2 dx,$$

justifying the procedure.