

Advanced Mathematical Methods for Engineers - June 23 2016

1. Determine the general solution of the linear homogeneous system

$$\underline{z}' = \mathbb{A}\underline{z}, \quad \text{where} \quad \mathbb{A} = \begin{bmatrix} 4 & -3 \\ 3 & 2 \end{bmatrix}.$$

2. Consider the sequence $f_n : [-10, 10] \rightarrow \mathbb{R}$ defined by $f_n(x) = x e^{-n|x|}$. Compute the limit of the sequence $\{f_n\}$ in $C^0([-10, 10])$ and in $L^1(-10, 10)$.
3. Consider the Cauchy Problem

$$\begin{cases} y' = \frac{3y - 2x}{2y - 3x}, \\ y(x_o) = y_o. \end{cases}$$

Determine the main properties of its general solution and draw a qualitative graph, as (x_o, y_o) ranges in a proper open set $D \subseteq \mathbb{R}^2$.

4. Prove that

$$u = (2x - 3) \text{pv} \frac{1}{3x - 2}$$

is a tempered distribution and compute its Fourier transform, justifying all the steps.