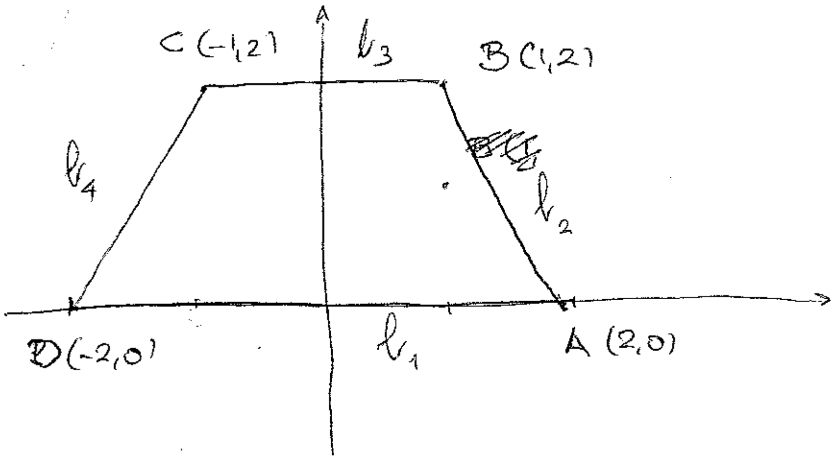

$Df = (2,1)$ non si annulla mai

$l_1: \quad y = 0 \qquad -2 \leq x \leq 2$

$l_2: \quad y = -2(x-2) \qquad 1 \leq x \leq 2$

$l_3: \quad y = 2 \qquad -1 \leq x \leq 1$

$l_4: \quad y = 2(x+2) \qquad -1 \leq x \leq 2$

$f|_{l_1} = 2x \qquad m_1 = -4 \qquad M_1 = 4$

$f|_{l_2} = \cancel{2x} - \cancel{2x} + 4 = 4 \qquad m_2 = M_2 = 4$

$f|_{l_3} = 2x + 2 \qquad M_3 = 4 \qquad m_3 = 0$

$f|_{l_4} = 2x + 2x + 4 = 4x + 4 \qquad M_4 = 0 \qquad m_4 = -4$

$M_{ass} = \max\{4, 4, 4, 0\} = 4$

$m_{ass} = \min\{-4, 4, 0, -4\} = -4$

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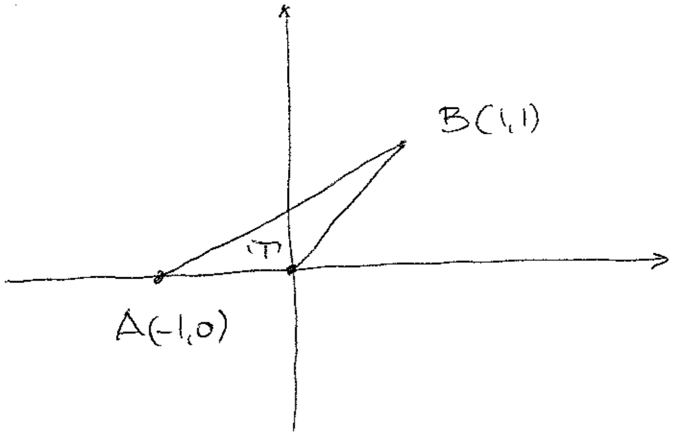
② $S: \quad z = 3x - \sqrt{6}\,y + 2 \qquad z_x = 3$

$(x,y) \in T \qquad z_y = -\sqrt{6}$

$$dS = \sqrt{1 + z_x^2 + z_y^2}\, dx\,dy$$

$$= \sqrt{1 + 9 + 6}\; dx\,dy = 4\, dx\,dy$$



$$T: \begin{cases} 0 \le y \le 1 \\ 2y - 1 \le x \le y \end{cases}$$

Quindi

$$\iint_{\Sigma} y^2 dS = 4 \int_0^1 \int_{2y-1}^{y} y^2\, dx\,dy = 4 \int_0^1 y^2 (y - 2y + 1)\, dy$$

$$= 4 \int_0^1 y^2 (1 - y)\, dy = 4 \left[\int_0^1 y^2 dy - \int_0^1 y^3 dy \right]$$

$$= 4 \left. \frac{y^3}{3} \right|_0^1 - 4 \left. \frac{y^4}{4} \right|_0^1 = \frac{4}{3} - 1 = \frac{1}{3}.$$

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(3) $\lambda^3 + 2\lambda^2 - 3\lambda = 0$

$\lambda (\lambda^2 + 2\lambda - 3) = 0$

$\lambda (\lambda + 3)(\lambda - 1) = 0$

$\lambda_1 = 0$

$\lambda_2 = -3$

$\lambda_3 = 1$

$$y = C_1 + C_2 e^{-3x} + C_3 e^{x} + Ax^3 + Bx^2 + Cx$$

$$y_p = Ax^3 + Bx^2 + Cx$$

$$y_p' = 3Ax^2 + 2Bx + C$$

$$y_p'' = 6Ax + 2B$$

$$y_p''' = 6A$$

$$\begin{cases} 6A \\ + 4B \quad + 12Ax \\ - 3C \quad - 6Bx \quad - 9Ax^2 = 9x^2 \end{cases}$$

$$\begin{cases} -9A = 9 \\ 12A - 6B = 0 \\ 6A + 4B - 3C = 0 \end{cases}$$

$A = -1$

$B = -2$

$-6 - 8 = 3C \qquad C = -\frac{14}{3}$

$$y = C_1 + C_2 e^{-3x} + C_3 e^x - x^3 - 2x^2 - \frac{14}{3}x$$

$$y' = -3C_2 e^{-3x} + C_3 e^x - 3x^2 - 4x - \frac{14}{3}$$

$$y'' = +9C_2 e^{-3x} + C_3 e^x - 6x - 4$$

$$\begin{aligned} y(0) &= \\ y'(0) &= \\ y''(0) &= \end{aligned} \begin{cases} C_1 + C_2 + C_3 = \frac{7}{2} \\ -3C_2 + C_3 = -\frac{14}{3} + \frac{14}{3} \\ 9C_2 + C_3 = -4 + 4 \end{cases}$$

$$\begin{cases} C_1 + C_2 + C_3 = \frac{7}{2} \\ -3C_2 + C_3 = 0 \\ 9C_2 + C_3 = 0 \end{cases} \qquad \begin{aligned} C_1 &= \frac{7}{2} \\ C_2 &= C_3 = 0 \end{aligned}$$

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(4) $I = \{ x \in \mathbb{R} : -1 \le x + 1 \le 1 \} = \{ x \in \mathbb{R} : -2 \le x \le 0 \}$

La serie converge anche negli estremi, come è immediato verificare sostituendo. Infatti:

$x = 0 \qquad \sum_{n=0}^{\infty} \frac{(-)^n}{n+2}$ converge per il criterio di Leibniz

$x = -2 \qquad \sum_{n=0}^{\infty} \frac{(-)^n}{n+2}$ come sopra

Per il Teorema di Abel, la serie converge uniformemente in tutto I.

$$\frac{\partial f}{\partial x}(2,3) = \lim_{h\to 0} \frac{f(2+h,3) - f(2,3)}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2 3 - 4\cdot 3}{h} = 3 \lim_{h\to 0} \frac{\cancel{4}+4h+h^2 - \cancel{4}}{h}$$

$$= 12$$

$$\frac{\partial f}{\partial y}(2,3) = \lim_{k\to 0} \frac{f(2,3+k) - f(2,3)}{k}$$

$$= \lim_{k\to 0} \frac{4(3+k) - 4\cdot 3}{k} = 4$$

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(6) $$\int_\ell \sqrt{6x+y^2+z^2}\, d\sigma_1 = \int_1^2 \sqrt{\cancel{6}^3 \cdot \frac{3t^2}{\cancel{2}} + 4\sin^2 t + 4\cos^2 t}\,\cdot$$

$$\cdot \sqrt{9t^2 + 4\cos^2 t + 4\sin^2 t}\, dt = \int_1^2 \sqrt{9t^2+4}\cdot\sqrt{9t^2+4}\, dt$$

$$= \int_1^2 (9t^2+4)\,dt = \left[\cancel{9}^3 \frac{t^3}{\cancel{3}} + 4t\right]_1^2 = 3(8-1) + 4(2-1) = 25$$

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(7) $f \in C^\infty(\mathbb{R}^2)$ ed è, dunque, ovunque differenziabile

$$f(0,\tfrac{1}{4}) = \operatorname{arctg}(0 + 4\tfrac{1}{4}) = \operatorname{arctg} 1 = \frac{\pi}{4}$$

L'equazione del piano tangente è

$$z - \frac{\pi}{4} = f_x(0,\tfrac{1}{4})x + f_y(0,\tfrac{1}{4})(y - \tfrac{1}{4})$$

Poichè

$$f_x = \frac{2}{1+(2x+4y)^2} \qquad f_x(0,\tfrac{1}{4}) = \frac{2}{2} = 1$$

$$f_y = \frac{4}{1+(2x+4y)^2} \qquad f_y(0,\tfrac{1}{4}) = \frac{4}{2} = 2$$

includiamo ~~che~~

$$z - \frac{\pi}{4} = x + 2\left(y - \frac{1}{4}\right).$$

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(8) $e^{-2xy} + xz^2 + y^2 z - 2 = 0$

$$f \in C^\infty(\mathbb{R}^3) \implies f_1, f_2 \in C^0(\mathbb{R}^3)$$

Inoltre

$$f(0,-1,1) = 1 + 0 + 1 - 2 = 0$$

$$f_z = 2xz + y^2 \qquad f_z(0,-1,1) = 1 \neq 0$$

Il Teorema di Dini è dunque verificato e $f(x,y,z)=0$ definisce $z = g(x,y)$ in un intorno di P.

Infine

$$\nabla g = \left(-\frac{\partial f/\partial x}{\partial f/\partial z}, -\frac{\partial f/\partial y}{\frac{\partial f}{\partial z}}\right)$$

$$\frac{\partial f}{\partial x} = -2y\, e^{-2xy} + z^2 \qquad \frac{\partial f}{\partial x}(0,-1,1) = 2 + 1 = 3$$

$$\frac{\partial f}{\partial y} = -2x\, e^{-2xy} + 2yz \qquad \frac{\partial f}{\partial y}(0,-1,1) = -2$$

$$\nabla g = \left(-\frac{3}{1}, -\frac{-2}{1}\right) = (-3, 2)$$

$$\frac{\partial g}{\partial v}(0,-1) = \langle \nabla g, \underline{v} \rangle = \langle (-3,2), (\tfrac{3}{5}, \tfrac{4}{5}) \rangle$$

$$= -\frac{9}{5} + \frac{8}{5} = -\frac{1}{5}.$$