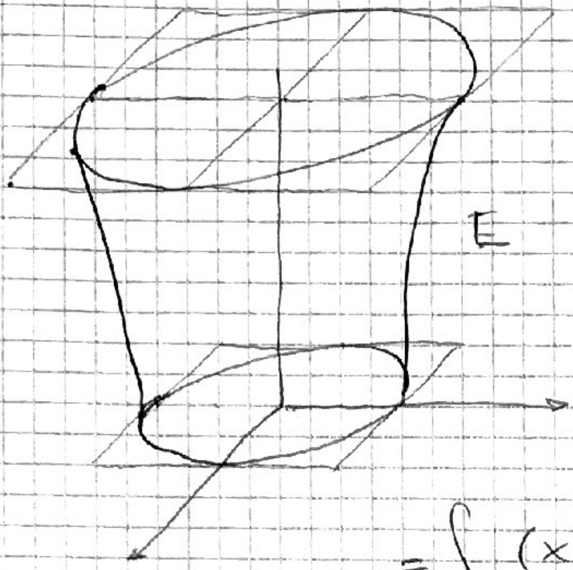
(1) $F = \left(\frac{x^3}{3} + 2z, \ \frac{y^3}{6} + 2x, \ \frac{zy^2}{2} \right)$

$E = \{ (x, y, z) \in \mathbb{R}^3 : \quad x^2 + y^2 - z^2 \leq 1, \quad 0 \leq z \leq \sqrt{3} \}$



Per il Teorema della divergenza

$$\phi_E(F) = \int_E \operatorname{div} F \, dx\,dy\,dz$$

$$= \int_E \left(x^2 + \frac{y^2}{2} + \frac{y^2}{2} \right) dx\,dy\,dz$$

$$= \int_E (x^2 + y^2) \, dx\,dy\,dz$$

Osserviamo che

$$E = \{ 0 \leq z \leq \sqrt{3}, \quad x^2 + y^2 \leq 1 + z^2 \}$$

Integriamo per strati

$$\int_E \operatorname{div} F \, dx\,dy\,dz = \int_0^{\sqrt{3}} \left(\int_{(x^2+y^2) \leq 1+z^2} (x^2+y^2) \, dx\,dy \right)$$

$$= \int_0^{\sqrt{3}} \left(\int_0^{2\pi} d\theta \int_0^{\sqrt{1+z^2}} \rho^2 \rho \, d\rho \right) dz$$

$$= \int_0^{\sqrt{3}} 2\pi \left[\frac{\rho^4}{4} \right]_{\rho=0}^{\rho=\sqrt{1+z^2}} dz$$

$$= \frac{2\pi}{4} \int_0^{\sqrt{3}} (1+z^2)^2 \, dz = \frac{\pi}{2} \int_0^{\sqrt{3}} (1 + 2z^2 + z^4) \, dz$$

$$= \frac{\pi}{2} \left[z + \frac{2}{3} z^3 + \frac{1}{5} z^5 \right] = \frac{\pi}{2} \left[\sqrt{3} + \frac{2}{\cancel{3}} \cancel{3} \sqrt{3} + \frac{1}{5} 3 \cdot 3 \cdot \sqrt{3} \right]$$

$$= \frac{\pi}{2} \sqrt{3} \left(1 + 2 + \frac{9}{5} \right) = \frac{24}{5} \frac{\pi}{2} \sqrt{3} = \frac{12}{5} \pi \sqrt{3}$$

(2) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{2^n\, n!}$

Calcoliamo $\displaystyle\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{1}{2^{n+1}(n+1)!}}{\frac{1}{2^n\, n!}} =$

$\displaystyle = \lim_{n\to\infty} \frac{2^n\, n!}{2^{n+1}(n+1)!} = \lim_{n\to\infty} \frac{1}{2(n+1)} = 0$

Dunque $R = \infty$ e la serie converge in tutto $\mathbb{R}$.

Inoltre

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{2^n\, n!} = \sum_{n=0}^{\infty} (x-2) \frac{\left(\frac{x-2}{2}\right)^n}{n!} =$$

$$= (x-2)\exp\left(\frac{x-2}{2}\right)$$

Infine

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n = \sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{2^n\, n!}$$

con $x_0 = 2$. Pertanto, dovendo far corrispondere gli esponenti

$$\frac{f^{(27)}(2)}{(27)!} = \frac{1}{2^{26}(26)!}$$

$$f^{(27)}(2) = \frac{27}{2^{26}}$$

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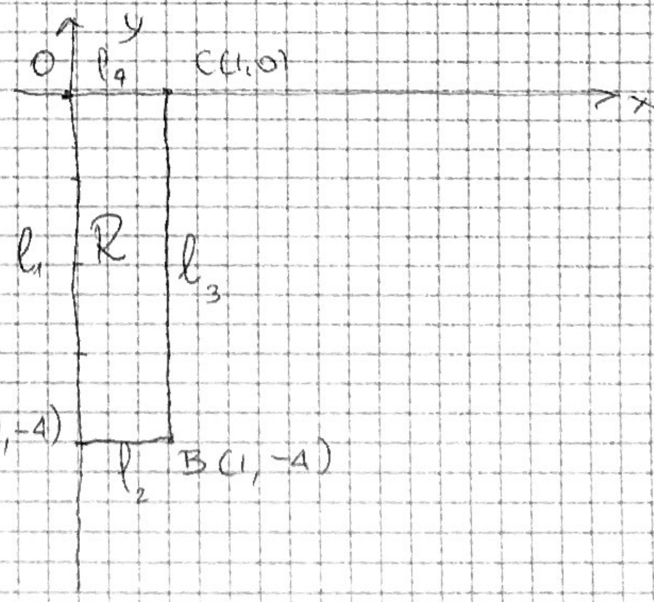
(3) $f(x,y) = \dfrac{(y+4)^2}{x-2}$



$f \in C^\infty(\mathbb{R}^2 \setminus \{x=2\})$

Applichiamo la <u>CN</u>

Abbiamo

$$\nabla f = 0 \qquad \begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \qquad \begin{cases} -\dfrac{(y+4)^2}{(x-2)^2} = 0 \\[2ex] \dfrac{2(y+4)}{x-2} = 0 \end{cases}$$

Concludiamo che $P_t\,(t,-4)$ sono tutti punti che soddisfano la CN. Tutti i punti sono sul bordo, per cui evitiamo di applicare la C.S.

Osserviamo che $f(t,-4) = \dfrac{(-4+4)^2}{t-2} = 0$

Abbiamo

$$l_1 : \begin{cases} x = 0 \\ y = s \end{cases} \quad -4 \leq s \leq 0$$

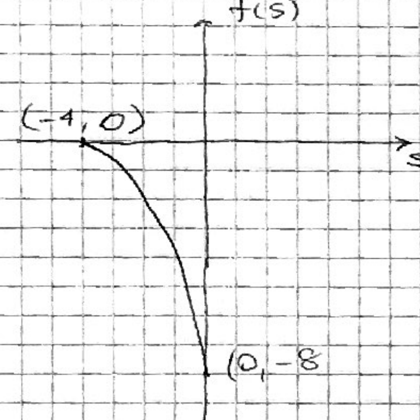
$$l_2 : \begin{cases} x = s \\ y = -4 \end{cases} \quad 0 \leq s \leq 1$$

$$l_3 : \begin{cases} x = 1 \\ y = s \end{cases} \quad s \in [-4,0]$$

$$l_4 : \begin{cases} x = s \\ y = 0 \end{cases} \quad s \in [0,1]$$

Inoltre

$$f|_{l_1} = + \frac{(s+4)^2}{-2} = -\frac{1}{2}(s+4)^2 \qquad s \in [-4,0]$$



$M_1 = 0$

$m_1 = -8$

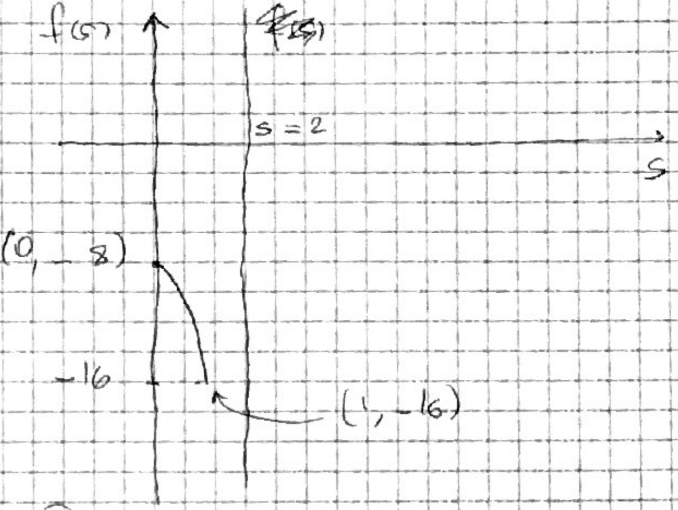
$$f|_{l_2} = 0 \qquad M_2 = m_2 = 0$$

$$f_{l_3} = \frac{(s+4)^2}{-1} = -(s+4)^2 \qquad s \in [-4,0]$$

$M_3 = 0$

$m_3 = -16$

$$f\big|_{\ell_4} = + \frac{16}{s-2} \qquad s \in [0,1]$$



$$M_4 = -8$$

$$m_4 = -16$$

Pertanto

$$M_{ass} = \max\{0, 0, 0, -8\} = 0$$

$$m_{ass} = \min\{-8, 0, -16, -16\} = -16$$

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(4)
$$\begin{cases} x(t) = t\cos t \\ y(t) = t\sin t \\ z(t) = \frac{2\sqrt{2}}{3} t^{3/2} \end{cases} \qquad t \in [0,1]$$

$$\begin{cases} x'(t) = \cos t - t\sin t \\ y'(t) = \sin t + t\cos t \\ z'(t) = \frac{\cancel{2}\sqrt{2}}{\cancel{3}} \frac{\cancel{3}}{\cancel{2}} t^{1/2} = \sqrt{2}\sqrt{t} \end{cases}$$

Pertanto

$$\int_\Gamma \frac{1}{x^2+y^2-4}\, d\sigma_1 = \int_0^1 \frac{1}{t^2-4} \sqrt{\cos^2 t + t^2 \sin^2 t - \cancel{2t \sin t \cos t} + t^2\cos^2 t + \sin^2 t + \cancel{2t\sin t\cos t} + 2t}\; dt$$

$$= \int_0^1 \frac{1}{t^2-4}\sqrt{1 + t^2 + 2t}\; dt = \int_0^1 \frac{t+1}{t^2-4}\, dt$$

$$= \int_0^1 \left(\frac{A}{t-2} + \frac{B}{t+2} \right) dt$$

Determiniamo A e B. Abbiamo $A(t+2) + B(t-2) = (t+1)$

$t = 2 \qquad 4A = 3 \qquad A = 3/4$

$t = -2 \qquad -4B = -1 \qquad B = 1/4$

Pertanto

$$= \int_0^1 \left(\frac{1/4}{t+2} + \frac{3/4}{t-2} \right) dt = \left[\frac{1}{4} \ln(t+2) + \frac{3}{4} \ln|t-2| \right]_0^1$$

$$= \frac{1}{4} \ln 3 + \cancel{\frac{3}{4} \ln 1} - \frac{1}{4} \ln 2 + \frac{3}{4} \ln 2$$

$$= \frac{1}{4} \ln 3 + \frac{1}{2} \ln 2$$

B2

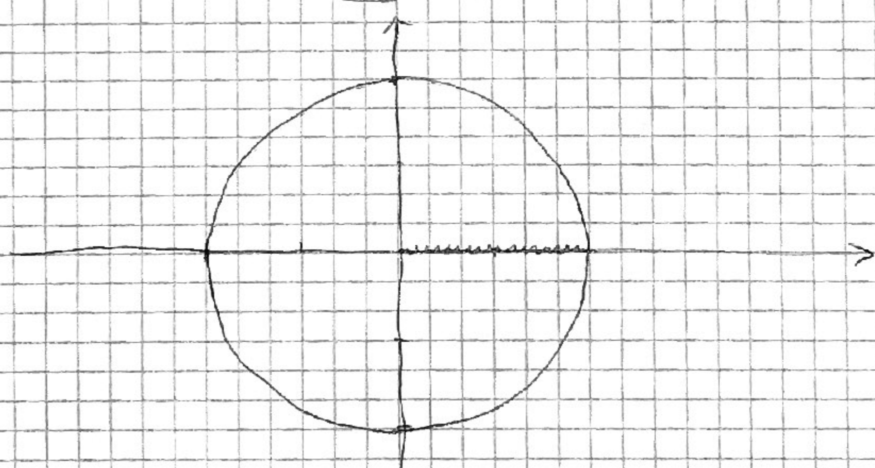
$$\begin{cases} x(t) = \cos^3 t \\ y(t) = \sin^3 t \end{cases} \qquad t \in \left[-\frac{\pi}{4}, \frac{\pi}{4} \right]$$

Le due funzioni $x(t)$ e $y(t)$ sono di classe C^∞.
L'insieme di definizione del parametro è un intervallo chiuso e limitato.

$$\begin{cases} x'(t) = -3 \cos^2 t \sin t \\ y'(t) = 3 \sin^2 t \cos t \end{cases}$$

Osserviamo che $x'(0) = y'(0) = 0$.
Pertanto <u>non</u> si tratta di un arco regolare

B3



Interno di Ω $= \{ x^2+y^2 < 4 \} \setminus \{ y=0,\ 0 \le x \le 2 \}$

Frontiera di $\Omega = \{ x^2+y^2 = 4 \} \cup \{ y=0,\ 0 \le x \le 2 \}$

Chiusura di $\Omega = \{ x^2+y^2 \le 4 \}$

B4

$$\frac{\partial}{\partial u} f(g_1(u,v), g_2(u,v)) = \frac{\partial f}{\partial x} \frac{\partial g_1}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial g_2}{\partial u}$$