A1 $\underline{F} = \left(rx + 4z,\ ry + 3x,\ \frac{z^2}{2} \right)$

Calcoliamo il flusso, utilizzando il Teorema della divergenza.
Perciò

$$\Phi_{\partial E}(\underline{F}) = \int_E \operatorname{div} \underline{F} \, dx\,dy\,dz$$

Osserviamo che

$$\operatorname{div} \underline{F} = 2r + z$$

$$E = \begin{cases} \frac{r}{4} \leq z \leq \frac{r}{2} \\ x^2+y^2 \leq r^2 - z^2 \end{cases}$$

Pertanto

$$\int_E \operatorname{div} \underline{F} \, dx\,dy\,dz = \int_{\frac{r}{4}}^{\frac{r}{2}} \int_{x^2+y^2 \leq r^2-z^2} (2r+z) \, dx\,dy\,dz$$

$$= \int_{r/4}^{r/2} (2r+z)\, \pi \,(r^2 - z^2)\, dz$$

$$= \pi \int_{r/4}^{r/2} \left(2r^3 + r^2 z - 2rz^2 - z^3 \right) dz$$

$$= \pi \left[2r^3 z + \frac{r^2 z^2}{2} - \frac{2r z^3}{3} - \frac{z^4}{4} \right]_{r/4}^{r/2}$$

A2 Consideriamo la serie $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(x-6)^{2n}}{5^n (2n)!}$

Poiché $5 = (\sqrt{5})^2$, possiamo riscrivere

$$\sum_{n=0}^{\infty} (-)^n \frac{(x-6/\sqrt{5})^{2n}}{(2n)!}$$

Osserviamo che

$$\cos t = \sum_{n=0}^{\infty} (-)^n \frac{t^{2n}}{(2n)!} \qquad \text{con} \quad R = \infty$$

Pertanto, con $R = \infty$

$$f(x) = \sum_{n=0}^{\infty} (-)^n \frac{(x-6/\sqrt{5})^{2n}}{(2n)!} = \cos\left(\frac{x-6}{\sqrt{5}}\right)$$

Inoltre

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(6)}{n!} (x-6)^n = \sum_{n=0}^{\infty} (-)^n \frac{(x-6/\sqrt{5})^{2n}}{(2n)!}$$

$$\frac{f^{(12)}(6)}{12!} = (-)^6 \left(\frac{1}{\sqrt{5}}\right)^{12} \frac{1}{12!}$$

$$f^{(12)}(6) = \frac{1}{5^6}$$

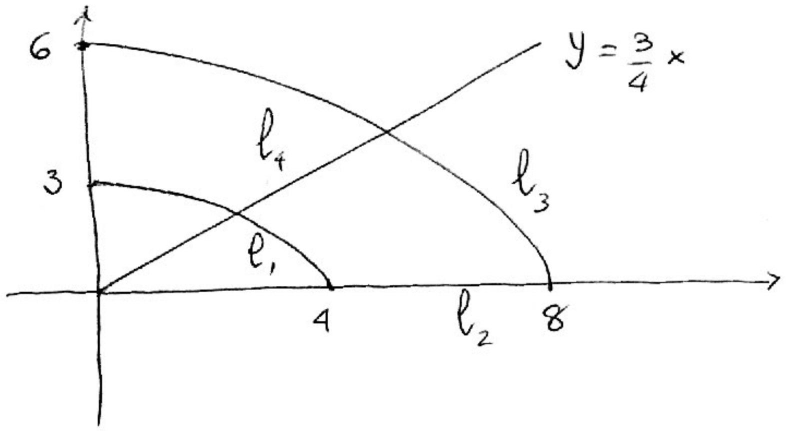
A3 $\quad f = 1 + 9x^2 + 16y^2$

Possiamo riscrivere

$$f = 1 + 144\left\{\frac{x^2}{16} + \frac{y^2}{9}\right\}$$

Il dominio è

$$K = \left\{ (x,y) \in \mathbb{R}^2 : \frac{x^2}{16} + \frac{y^2}{9} \geq 1, \quad \frac{x^2}{16} + \frac{y^2}{9} \leq 4 \right.$$
$$\left. 0 \leq y \leq \frac{3}{4} x \right\}$$



Osserviamo che

$$\ell_1 = \left\{ \frac{x^2}{16} + \frac{y^2}{9} = 1, \quad 0 \leq y \leq \frac{3}{4} x \right\} \qquad f\big|_{\ell_1} = 1 + 144 = 145$$

$$\ell_3 = \left\{ \frac{x^2}{16} + \frac{y^2}{9} = 4, \quad 0 \leq y \leq \frac{3}{4} x \right\} \qquad f\big|_{\ell_3} = 1 + 4 \cdot 144 = 577$$

La funzione, infatti, assume valore costante sulle ellissi

$$\frac{x^2}{16} + \frac{y^2}{9} = c \qquad c \in \mathbb{R}_+$$

Possiamo, dunque, concludere, senza applicare altri metodi, che

$$m_K = 145$$
$$M_K = 577$$

A4 $$\int_\Gamma z^{3/2} \, d\sigma_1 = \int_1^2 3^3 t^3 \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} \, dt$$

$$= \int_1^2 27t^3 \sqrt{1+324t^2}\, dt$$

$$= 27 \int_1^2 t^2 \sqrt{1+324t^2}\, t\,dt$$

Poniamo

$$1+324t^2 = s^2$$

$$t^2 = \frac{s^2-1}{324}$$

$$t\,dt = \frac{1}{324} s\,ds$$

$$t=1 \longrightarrow s = 325$$

$$t=2 \longrightarrow s = 1297$$

Pertanto otteniamo

$$\int_\Gamma z^{3/2}\, d\sigma_1 = \int_{325}^{1297} 27 \cdot \frac{s^2-1}{324} \cdot s \cdot \frac{s}{324}\, ds$$

$$= \frac{27}{(324)^2} \int_{325}^{1297} (s^2-1)s^2\, ds$$

$$= \frac{27}{(324)^2} \left[\frac{s^5}{5} - \frac{s^3}{3} \right]_{325}^{1297}$$

B1 $\quad \Gamma : \begin{cases} x = \mathrm{Ch}^3 t \\ y = \mathrm{Sh}^3 t \end{cases} \qquad t \in [-4,4]$

Osserviamo che

$$\begin{cases} x' = 3\,\mathrm{Ch}^2 t\ \mathrm{Sh}\,t \\ y' = 3\,\mathrm{Sh}^2 t\ \mathrm{Ch}\,t \end{cases}$$

Poiché $x'(0) = y'(0) = 0$, concludiamo che la linea <u>non</u> è regolare

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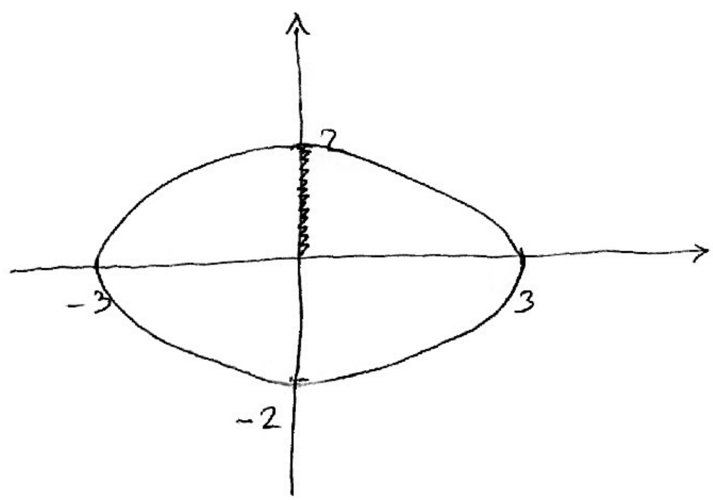
B3 $\quad \Omega = \{(x,y) \in \mathbb{R}^2 : 4x^2 + 9y^2 \leq 36\} \setminus \{(x,y) \in \mathbb{R}^2 : x = 0,\ 0 \leq y \leq 2\}$



$$\mathrm{Int}\,\Omega = \{4x^2 + 9y^2 < 36\} \setminus \{x = 0,\ 0 \leq y \leq 2\}$$

$$\partial\Omega = \{4x^2 + 9y^2 = 36\} \cup \{x = 0,\ 0 \leq y \leq 2\}$$

$$\overline{\Omega} = \{4x^2 + 9y^2 \leq 36\}$$

B4 $\quad \dfrac{\partial}{\partial v} f(g_1(u,v), g_2(u,v)) = \dfrac{\partial f}{\partial x}(g_1, g_2)\dfrac{\partial g_1}{\partial v} + \dfrac{\partial f}{\partial y}(g_1, g_2)\dfrac{\partial g_2}{\partial v}$